Astronomy Reports, vol. 39, no. 4, pp.489-497, 1995

The Effect of Opacity on Parameters of the Convective Envelope in Solar Models

V. A. Baturin and S. V. Ayukov

Sternberg Astronomical Institute, Universitetskii pr. 13, Moscow, 119899 Russia

Received June 23.1994

Abstract

High-precision calculations of a series of standard models of the internal structure of the Sun using different opacity tables are presented. The models are compared on the basis of two key parameters - helium abundance and specific entropy in the convection zone. The effect of opacity on these two parameters in the radiative interior of the Sun is studied. Comparison of standard models calculated using different opacity tables shows that the specific entropy in the envelope diminishes with growing opacity in the radiative zone, while helium abundance grows with increase in opacity in the core. Investigation of static nonevolutionary models with modified opacities indicates a general connection between the specific entropy in the envelope and opacity in the radiative zone, outside the region of energy release.

INTRODUCTION

The classical problem of modelling the internal structure of the Sun has not lost its urgency due, on the one hand, to continuous revision of theoretical ideas about the physics of high-temperature plasmas and, on the other hand, to the rapid development of helioseismology, which requires high-precision calculations of the distributions of physical quantities in the model. In the 1990s, a transition is taking place to a new generation of Solar models, distinguished most of all by their high internal mathematical consistency, which allows not only identification and study of small effects of different physical assumptions, but also analysis of internal relations between parameters in the problem of modelling the Solar interior.

In this paper, we first consider "standard" evolution ary Solar models, and remain within the framework of classical evolutionary ideas. This means that the basis of the calculation is the concept of equilibrium (thermodynamic) evolution from an initial uniform state to a model with the present-day radius and luminosity for a given age. We assume mass conservation during the process of evolution (no accretion or mass loss) and the absence of any kind of mixing (diffusive or other) outside the convection zone. We assume conversion of hydrogen to helium via three $(p-p)$ chains and the CNO cycle, during which all intermediate components of these reactions are assumed to be in kinetic equilibrium; the question of the initial and final concentrations is not considered. Terms corresponding to energy release in quasistatic evolution were included in the equation for luminosity (i.e., in the law of conservation of energy), but no dynamic effects were taken into account in the structure of equations for individual models (i.e., the dynamic effects of convection and the influence of rotation are ignored). Every model is assumed to be spherically symmetrical and in hydrostatic equilibrium under the action of self-gravitation and the pressure of matter and radiation.

For these assumptions about evolution, to construct a model of the present-day Sun we need only the profile of chemical composition X, which is the only evolving parameter of the problem. By "chemical composition" we will mean the abundance of hydrogen X (or helium Y), since the fraction of heavy elements Z in our calculations was fixed ($Z = 0.02$). We will call a model constructed under the assumption above standard, or evolutionary. However, taking into account possible errors in the evolutionary considerations, it is admissible to study models with nonevolutionary chemical composition profiles, requiring that these models have the Solar values of radius, luminosity, and mass. Such models may be called nonstandard, or static. The second part of this paper treats nonstandard Solar models with variations of opacity and chemical composition.

The Vogt-Russel theorem [1] asserts that for a given profile of chemical composition (either evolutionary or not) and mass, the internal structure of a star is fully determined. This implies that the model will have a definite radius and luminosity, and in order for these to have the Solar values, there must be at least two free parameters. During construction of a standard model. the parameters usually selected for this are the initial hydrogen abundance (which coincides with the hydrogen abundance in the model envelope for the evolutionary assumptions made) and a certain parameter of the convection theory, allowing control of the temperature gradient in the outermost layers of the convection zone. The choice of these parameters is to a considerable extent historical and due to lack of reliable independent estimates for these quantities. In other words, these parameters turn from arbitrarily chosen quantities into the final result of the problem solution. For example, the standard-model helium abundance is the basic (and the most precise) estimate of the Solar-system helium abundance used in astrophysics [2]. In the general case, the neutrino flux, convection zone depth, and characteristics of the oscillation spectrum may be considered results of the solution. However, it is important to keep in mind that when constructing a model, in addition to the hydrogen-abundance profile, at least four *a priori* functions are used, describing energy release, opacity, convective gradient, and the equation of state; these functions are known to some, often not very high, accuracy. Therefore, the actual problem of the internal structure of the Sun is a matter of determining the errors of the resulting parameters, depending on variations of *a priori* physical functions.

In this paper, we consider some aspects of this problem. First, we attempt to overcome the drawbacks of using the convection parameter as a model parameter by introducing instead the specific entropy in the adiabatic part of the convection zone. Second, we consider the effect of change of the opacity tables used on the parameters of the Standard Solar Model (SSM). Third, we consider relationships between Solar model parameters (helium abundance, entropy, and convection zone depth), depending on variations of opacity and chemical composition in nonevolutionary Solar models.

STANDARD SOLAR MODELS

The general physical assumptions used in calculations of standard models were described in the Introduction. We list here the main parameters used in the calculations: the Solar mass $M_{\odot} = 1.989 \cdot 10^{33} g$, luminosity $L_{\odot} = 3.846 \cdot 10^{33}$ erg/s [3], and apparent radius $R. = 6.9599 \cdot 10^{33}$ cm [4]. In the model, the apparent radius corresponds to optical depth $\tau = 0.01$. The assumed age of the Sun ($t_{\odot} = 4.75$ billion years) is based on the ages of the oldest meteorites (although the latest data yield a somehow lower value -4.55 billion years [5,6], we used the more conventional age when modelling the internal structure). The calculation of every model in the evolutionary sequence consists of solution of a boundary-value problem for a set of fourth-order nonlinear differential equations describing hydrostatic equilibrium, conservation of energy, and the heat transfer process (see, for example, [1]). The interior boundary conditions correspond to the choice of the solutions regular at the center. The exterior boundary conditions actually reduce to an independent problem of calculation of the atmosphere and exterior convective layers, since, from the viewpoint of modelling of the internal structure, it is enough to require that the temperature and density approach zero and to determine the convection zone entropy. To calculate the structure of the atmosphere, an analytic approximation for temperature as a function of optical depth [7], parametrized for various luminosities was used. The integration started in the photosphere from point ($T = 10^{-4}$, $\rho = 10^{-9}$) with fitting of the apparent radius to the level $\tau = 0.01$, and was constructed using the dependence $T(\tau)$ up to the onset of convection, determined from the Schwarzschild criterion (the logarithmic gradient of temperature $\nabla = d\ln T/d\ln P$ greater than the adiabatic gradient). Atmosphere opacity, used in the relationship between density and optical depth, is essential for determination of the model radius.

The energy transfer in the convection zone was calculated using mixing-length theory [1]. All relevant thermodynamic quantities are interpolated from tables computed in the framework of a thermodynamic model of the Mihalas-Hummer-Däppen equation of state [8-10]. Thermonuclear reaction rates were calculated using the cross-sections presented in [11].

Special attention was paid to the numerical accuracy of the calculations. The accuracy of functions of the main variables (temperature T, radius r, density ρ, fractional mass M_r, fractional luminosity L_r) may be considered to be a general criterion of accuracy. Our software ensures relative precision not worse than 10^{-5}. The main mathematical features of the method used are the following: When solving the boundary-value problem for the structure of individual models, we used a ranging method. We integrated the solutions using a fourth-order Runge-Kutta method with variable step with the number of steps roughly 1700. Using $\mu = (M_r/M_\odot)^{2/3}$ as an independent variable permits avoidance of problems of the expansion at the center (and the occurrence of infinite derivatives; the expansion size $\mu_0 = 10^{-5} - 10^{-6}$). The main source of errors accumulated in the process of integrating the solutions by a high-order method toward the joining point is the nonsmoothness of the interpolated opacities. To reduce this effect, we used spline interpolation of the tabulated values of opacities and the equation of state. The accuracy of joining the solutions in the main variables is 10^{-6}. We chose the joining point to be at the level $\mu = 0.81 (r/R_\odot \approx 0.35)$ in all the models. To represent the hydrogen abundance profile $X(\mu)$, we used an auxiliary grid of 500 points, evenly spaced in μ. This provides sufficient accuracy of the interpolated values for $X(\mu)$ for the subsequent model. Calculation of evolution was conducted using a second-order explicit method with 80 equal time steps. The fit of the model radius and luminosity to the present-day Solar values was made with accuracy $3 \cdot 10^{-6}$.

Use of high-order integration methods and small steps does not guarantee the absence of errors in the software, which contained up to 300 kilobytes of text. To increase reliability of the code used (called MUSEC), we tested it using analytical functions for the opacity, energy release, and equation of state, and compared independent calculations of several groups in the framework of the international GONG project. Comparison using the main variables confirmed the accuracy of the solutions obtained at a level of 10^{-5}

[12].

When considering standard Solar models, we will concentrate on the effect of opacity on the resulting parameters. We used three types of opacity tables, computed over the last 20 years. First are the tables of Cox and Stewart [13] (henceforth CS70). These are rather old calculations, frequently used in different works and subsequently improved. Second, we used tables of the Los Alamos Opacity Library [14] (hence forth LAOL) for the heavy-element mixture of Ross and Aller [15], popular in the 1970s. The tables used were WKM2 and WKMIO, in the notation of [14]. Note that they were computed with a new version of the Los Alamos project programs, and are somewhat different from the frequently used LAOL opacities that are quoted in [16]. Third, we used new opacities, computed in the framework of the OPAL project and published by Iglesias and Rogers [17]. To avoid extrapolation for relatively low temperatures near the base of the convection zone, we supplemented these last tables with data from [18]. The tables were computed for the heavy-element mixture of Grevesse [19]. However, in this paper we do not analyze the influence of heavy-element abundance variations, since this is an intrinsic parameter of the tables, as well as of the computation algorithm; we use only the resulting values from the published tables for demonstration of the variations of model parameters. Since the LAOL and OPAL tables have a lower temperature limit, they were supplemented with the tables of Cox and Tabor [20] (hence forth CT76), with smooth joining of the tables in the region with temperature $(0.8 - 1.0) \cdot 10^6$ K in the convection zone, where the role of opacity is insignificant.

Figure 1 compares opacities under conditions typical for the radiative zone below the convection zone in the standard Solar model. It can be seen that each new table usually gave K values greater than the previous ones. Note that the transition from CS70 to LAOL corresponded to an increase of opacity for all temperatures, including the core. The transition from LAOL to OPAL mainly changed the opacities outside the energyreleasing core.

ENTROPY AS A SOLAR-MODEL PARAMETER

As we have already mentioned in the Introduction, the standard Solar model is characterized by two calibration parameters: the initial chemical composition and the convection parameter controlling the temperature gradient in the outer convective layers. The choice of these two quantities is to a considerable extent governed by historical and technical reasons, since, given these parameters, evolutionary tracks may be calculated. However, if determination of the helium abundance has independent meaning and interest, we actually do not use the information contained in the second parameter.

Nevertheless, to specify a model, both parameters are needed, the second being of even greater importance. If there is an extended convective envelope (as in the Sun) in which matter is adiabatically stratified, the requirement of zero exterior boundary conditions ($T \longrightarrow 0$, $p \longrightarrow 0$) is not sufficient, since it is fulfilled on any adiabat. Therefore, the entropy of the convective

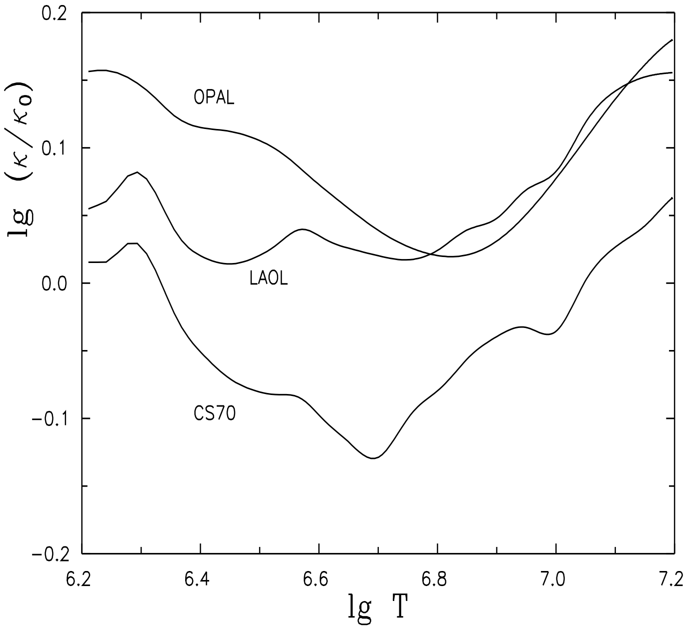

Figure 1: Comparison of different opacity tables under the conditions characteristic for the SSM radiative core. Differ ence between opacity logarithms and the simple analytic κ function [21] plotted as a function of temperature logarithm. The points $(T, \rho, X)($ for opacity calculations are taken from the model profile calculated with the OPAL opacities. Curves are labeled with the opacity table types.

adiabat must be specified, as a natural physical parameter of the structure (in early works on Solar modelling, quantities other than entropy were used, such as the polytropic constant, however it is clear that any quantity that is conserved on an adiabat is a function of the entropy). The convection parameter determining the temperature gradient in the region with superadiabatic convection and, hence, the choice of the appropriate adiabat, can be considered only as a technical parameter, since its value depends not only on the assumed theory of convection and its internal parameters, but also on the structure of the atmosphere, in particular, on the opacities used, on which depends the density at the upper boundary of the convection zone.

In actuality, using the convection parameter as a resulting parameter of the model not only makes it impossible to compare standard Solar models, but also is physically inadequate in the SSM problem. As is discussed in detail in [22], to construct a Solar model, it is necessary to conserve the entropy of the Solar envelope, while the structure of the outermost layers may be completely different. This means that we cannot determine the structure of the outer convective layers in the framework of the SSM, since in it only the thickness of the superadiabatic convection region and, with a very large relative uncertainty, the mass of these layers are determined.

In most work on Solar modelling, the convection zone entropy is not presented, nor are data sufficient to calculate it. At the same time, its calculation is not much more complicated than that for any other thermo dynamic quantity. The entropy difference between points (T, ρ, X) can easily be calculated independently of the integration path as

$$S = S_0(X) + \int\limits_{(T_0,\rho_0)}^{(T,\rho)} \frac{1}{T}\left[\left(\frac{\partial U}{\partial \rho}\right)_T - \frac{P}{\rho^2}\right] d\rho + \frac{C_v dT}{T}, \qquad (1)$$

(U is the internal energy and C_v is the specific heat capacity at constant volume). The only complications are the origin of coordinates and the chemical composition dependence. In our calculations, we used additional data for entropy at a fixed point ($T_0 = 15 \cdot 10^6 K, \rho_0 = 150 g/cm^3$) for different chemical compositions, kindly provided by W.Däppen. A simple analytical approximation of these data (at the point indicated above) is

$$S_0/R_g = -3.426523X^2 + 12.36082X + 6.739551 mol/g \qquad (2)$$

and is accurate to $\sim 10^3$ for $X \in [0.30 - 0.75]$ (the values of absolute specific entropy correspond to calculations for the Mihalas-Hummer-Däppen equation of state divided by the universal gas constant R_g). For calculations of entropy using statistical distributions, there is no arbitrary constant in its definition. The only problem is comparing the entropy for various models computed with different equations of state. However, the behavior of the

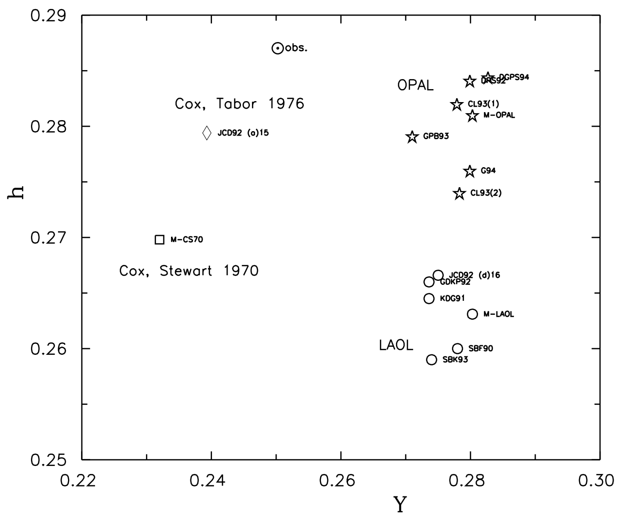

Figure 2: Dependence of SSM parameters on opacities used. Convection zone depth h is plotted as a function of helium abundance in the envelope Y. Different symbols designate models for each of the four opacity table types, and are labeled in accordance with Table 2. Models labeled M are listed in Table 1.

hot Solar plasma is rather close to that of an ideal gas; therefore, in Solar models, numerical differences in the entropy values are not great, being of the same order as differences in pressure, internal energy, etc. (about 0.1%). Thus, we will use and analyze the model structure from the viewpoint of helium abundance and convection zone specific entropy.

EFFECT OF OPACITY ON SSM PARAMETERS

Table 1 presents the main parameters of the standard Solar models computed for various opacities. Let us first consider the effect of opacity on SSM parameters from the viewpoint of helium abundance and convection zone depth h, since this will allow us to use for analysis data from numerous recently calculated models. In addition, the convection zone depth is logically related to the convection zone entropy for the given opacities. This quantity is of independent interest as well because there is an estimate of h from the observed frequencies of the five-minute oscillations (0.287 ± 0.003)

[23] and a compatible estimate [24]). This means that in order to bring such frequencies into agreement with the model frequencies, it is necessary for the model to have the indicated convection zone depth.

Figure 2 shows the location of the models constructed in the $(Y-h)$ plane. This figure shows some models published in the 1990s (see data and references in Table 2), computed with the same (or similar) opacities and equation of state.

Grouping of the models in accordance with the opacities used is obvious. During the transition from CS70 to LAOL opacities, the helium abundance considerably increased (by almost 5%) in connection with opacity increase in both the core and the radiative zone. The transition from LAOL to OPAL affected Y only weakly, since the opacity difference is small in the central parts of the star.

Table 1. Standard Solar models

Model	Opacity	Parameters		Central value		
		Y_0	S/R_g	$T_c/10^6$	ρ_c	X_c
M-CS70	CS70	0.2320	21.72	15.0549	149.413	0.3862
M-LAOL	LAOL	0.2803	21.05	15.6234	149.642	0.3379
M-OPAL	OPAL	0.2803	20.87	15.7172	150.843	0.3322
Obser.		0.2502	21.03			0.287
		±0.004	±0.08			

Model	Convective zone		ρ_b	Neutrino fluxes,SNU		
	h	$T_b/10^6$		^{37}Cl	^{71}Ga	^{115}In
M-CS70	0.2698	1.9984	0.1432	4.005	106.6	577.7
M-LAOL	0.2631	2.0025	0.1385	7.978	125.4	615.1
M-OPAL	0.2810	2.1907	0.1772	8.508	127.4	618.6
Obser.	0.287			2.33	83	
	±0.003				±19	

On the other hand, the transition from CS70 to LAOL led to a decrease of the convection zone depth, because of the helium abundance increase in the shell. The effect of enhanced helium abundance exceeded both that of increased opacity and of decreased entropy (both of these effects result in an increased convection zone depth. Fig. 3). Finally, using OPAL instead of LAOL lead to an increase of the convection zone depth (approaching the helioseismic determination), due to both increased opacity and decreased entropy (Fig. 3).

Estimates of the neutrino fluxes (Table 1, neutrino absorption cross-sections taken from [35]) in the standard models offer no surprises. The small fluxes in model M-CS70 are due to lower central temperatures, which are a consequence of lower opacities in the central regions.

We can formulate the general conclusion as follows: a model calculated on the basis of modern ideas about the core opacity (i.e., close to LAOL or OPAL) must have initial helium abundance $Y = 0.27-0.28$. Possible opacity variations (e.g., a decrease due to additional heat conduction mechanisms, such as WIMPs - see one of the latest reviews on this problem [36]) or collective phenomena [37] must result in a decrease of Y by 1% for decrease in K by 5%. Similarly, a decrease in opac ity leads to a decrease of the neutrino flux; however, the chlorine neutrino flux is quite sensitive to temperature and opacity at the center of the model, whereas Y depends on the opacity in a broader region of the radiative core. Thus, although there is a general correlation between the neutrino flux and the initial helium abundance (this was stressed for examples of standard and nonstandard models in [38,39]), this relationship is not unique.

The availability of numerous published models permits estimation of the effect of the equation of state on the SSM. At present, the Mihalas-Hummer-Däppen equation of state is the most complete and detailed. The most substantial physical effect of the equation of state is the interaction between charged plasma particles, leading to a deviation from ideal gas laws. Simple estimates show that accounting for electrostatic corrections reduces the pressure in the central parts of the Sun by $1 - 2\%$, which should affect SSM parameters. It is most simple to trace the effect of the equation of state for models computed with and without corrections.

Table 2. Some recent Solar models

Model	Reference	Equation of state	Opacity tables	Y	h	S/R_g	Notes
15	[25]	MHD	CT76	0.2393	0.2794		
14	[25]	CEFF	CT76	0.2404	0.2784	21.547	*
4	[25]	EFF	CT76	0.2496	0.2769	21.443	
16	[25]	MHD	LAOL	0.2750	0.2666		
13	[25]	EFF	LAOL	0.2849	0.2637		
22	[25]	EFF	LAOL	0.2747	0.2729	21.007	$t_\odot = 4.5 \times 10^9 y.$
6	[25]	EFF	LAOL	0.2776	0.2696	21.009	$t_\odot = 4.75 \times 10^9 y.$
21	[25]	EFF	LAOL	0.2800	0.2667	21.012	$t_\odot = 5.05 \times 10^9 y.$
7	[25]	EFF	LAOL	0.2806	0.2712	20.952	
27	[25]	EFF	LAOL	0.2720	0.2790	20.982	**
28	[25]	EFF	LAOL	0.2650	0.2857	21.001	***
DPS92	[26]	MHD	OPAL	0.2799	0.2841		
DGPS94	[27]	MHD	OPAL	0.2827	0.2844		
KDG91	[28]	MHD	LAOL	0.2736	0.2645		
SBF90	[29]		LAOL	0.278	0.260		
CL93(1)	[30]	MHD	OPAL	0.2779	0.282		
CL93(2)	[30]	MHD	LAOL	0.2783	0.274		
GPB93	[31]		OPAL	0.271	0.2791		
G94	[32]		OPAL	0.2799	0.276		
GDKP92	[33]	MHD	LAOL	0.2736	0.266		
SBK93	[34]		LAOL	0.2740	0.259		

Notes: * CEFF is the equation of state of Eggleton, Faulkner, and Flannery [41] corrected for Coulomb interaction of atoms.

** Core opacity reduced by a factor of 0.2.

** Core opacity reduced by a factor of 0.4. Such models were described in [25,

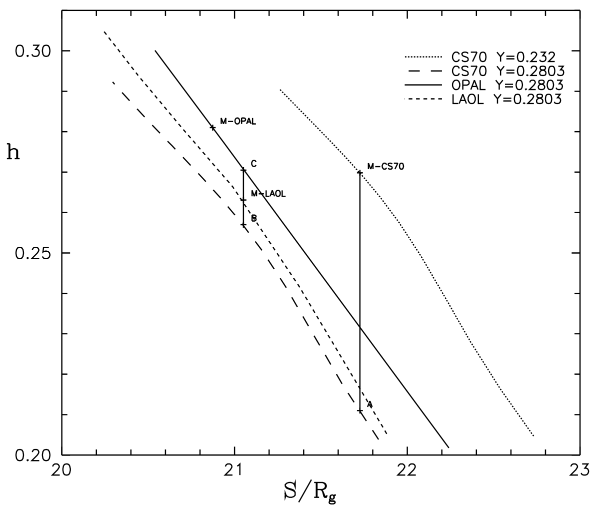

Figure 3: Dependence of convection zone depth h on specific entropy for three types of opacities and helium abundances in the SSM calculated. Transition from point M-CS70 to point A illustrates change in h due to helium abundance increase for fixed entropy and opacity tables; transition from A to B illustrates change in h due to change in entropy; and from B to M-LAOL, due to the opacity increase itself between the tables CS70 and LAOL. Similarly, the transition from M-LAOL to C corresponds to change in opacity between the tables, and from C to M-OPAL, to increase in entropy.

30, 33]. Variations of the helium abundance and the convection zone depth are approximately the same in all of these papers. Therefore, to study variations of entropy in the models, we used the data on models from [25] (models 4 and 14 in Fig. 4). We found that accounting for electrostatic interaction results in a decrease of helium abundance by 1% and an increase of entropy, with modest increase of the convection zone depth (by 0.25%). In addition, a comparison between models computed with the ideal gas equation of state plus electrostatic corrections (model 14) and Mihalas-Hummer-Däppen models (model 15) shows that introduction of the corrections actually depletes the effect of the more complicated Mihalas-Hummer-Däppen equation of state on the SSM parameters.

STATIC (NONSTANDARD) SOLAR MODELS

Further, we consider the class of nonstandard Solar models, for which it is required only that they have the Solar values of radius, luminosity, and mass. To obtain the chemical composition profile in a model for the present-day Sun, we will use, instead of the evolution calculation, arbitrary $X(\mu)$ profiles obtained from the standard evolutionary model profile by means of two kinds of modifications. First, we admit modifications localized in the energy-releasing core, defining the new chemical composition by means of the profile $X_0(\mu)$ of the standard Solar model as

$$X(\mu) = X_0(\mu) + \delta X(X_{surf} - X_0(\mu)), \tag{3}$$

where δX is the modification parameter, and X_{surf} is the hydrogen abundance in the model envelope. It is obvious that the new profile will differ from the standard one only in regions where significant hydrogen burning takes place, i.e., for temperatures $> 10^7$ K. Such variations can to some extent mimic various complicated effects in Solar evolution, for example, weak or full mixing, diffusion, gravitational sedimentation of helium, and so forth. However, we do not specify the effects to which this may correspond (if any), but rather consider the core chemical composition profile as an unknown parametrized function.

Of course, the core profile behavior may be much more complicated than that described by (3). However, a comparison with certain pertinent numerical experiments [40] shows that the principal regularities and relationships become apparent even for such simple variations of the chemical abundance profile. Therefore, for sake of simplicity, we will henceforth assume $X(\mu)$ variations to be described by (3).

In addition, we consider more global variations of the chemical composition profile, assuming

$$X(\mu) = X_0(\mu) + \Delta X, \tag{4}$$

In this case, we will use the helium abundance in the envelope Y as a parameter for the set of profiles. The second class of variations is close to chemical composition profile variations between standard models computed with different opacities.

To reduce a model to the Solar radius and luminos ity, we need one more parameter, and will use the convection zone entropy. In this approach, there are enough free parameters to obtain a unique model if the remaining physical functions (opacity, equation of state, and energy release) are specified. Our aim is to study changes of specific entropy and helium abundance for opacity variations in parts of the Solar radiative zone, including the core (i.e., where opacities determine to a significant extent the behavior of model solutions and where opacity is one of the least well known quantities).

We represented opacity variations by functions peaked in the core, in the radiative zone itself, or at the convection zone base. The modified opacity has the form

$$\kappa = \kappa[1 + aexp(-(\log T - \log T_0)^2/D))], \tag{5}$$

where κ_0 is the initial opacity from the OPAL tables, a is the modification amplitude, $\log T_0$ is the location of the maximum correction, and D is the degree of limitation of the modification (in all cases $D = 0.01$). For modifications in the core, $\log T_0 = 7.2$, in the radiative zone, $\log T_0 = 6.6$, and near the convection zone base, $\log T_0 = 6.3$. The width of the variation region in the core is between 7.0 and 7.3 in $\log T$, or from the model center to radius 0.18; in the envelope, $6.4 < \log T < 6.8$ (radius $0.32 - 0.67$).

Using the core chemical composition modifications (3), we obtained the following results: opacity variations in the core do not lead to entropy changes in the envelope; opacity variations in the radiative zone consider ably affect the model entropy (for $a = 0.5$, the entropy change $\Delta S = -0.33$); for opacity variations at the convection zone base, the entropy changes are insignificant.

We can interpret these results in a different way. First, variations of opacity and of the X profile in the core influence the solution in the core in a similar way and cancel each other, since the envelope parameters (S and Y) do not change. This implies the possible exist ence of a set of models with similar envelopes but different cores due to variations of X and κ. However, at the same time this makes it impossible to construct a model for arbitrary envelope parameters at the expense only of variations of opacity and chemical composition profile in the core (see also [40]). Second, opacity variations near the convection zone base, which result in a considerable change of the convection zone depth, weakly affect the model parameters. This means that, if the possible errors of the opacities used is about 20%, the convection zone depth in the standard Solar model is determined to an accuracy of only 1%.

In the second case [profile change in accordance with (4)], the results are to a considerable extent consequences of the previous ones. In this case, we naturally must consider relationships in the (S_{cnv}, Y) plane, i.e., follow how a model moves when certain perturbations are imposed. Figure 4 shows the results for models labeled with crosses and circles (the standard OPAL model was the initial model). In this figure, the $S(Y)_\kappa$, dependence (the dependence for opacity modifications in the core is labeled $\delta\kappa$) is nearly parallel to the abscissa, i.e., the numerical value of entropy in the envelope changes little. This is to a considerable extent an accidental phenomenon associated with compensation of entropy variations with molecular weight and variation of the entropy differential in the convective envelope itself. However, of more importance is the fact that the $S(Y)_{chi}$ dependence for chemical composition profile variations in the core (the dependence for constant opacities is labeled δX) is close to $S(Y)_\kappa$. This is consistent with the above assertion that the effects of variations of opacity and chemical composition profile in the core are similar, since it means entropy is conserved in the core for other initial profiles with different Y.

On the other hand, opacity changes in the radiative zone [$a = 0.5$ in (4)] lead to changes of both entropy and helium abundance in the envelope. In the (S_{cnv}, Y) plane, the model moves from the point M-OPAL to the point M-OPAL', and parallel translation of the corre sponding $S(Y)$ relationships occurs (lines passing through the point M-OPAL'). In this case, the entropy changes are quantitatively consistent with the above estimates for chemical composition variations in the core. The exact displacement vector in the (S_{cnv}, Y) plane depends on the location of the maximum of the opacity perturbation function; the vector approaches the vertical and decreases its value as $\log T_0$ in (5) decreases.

Figure 4 shows as well models from [25] (their parameters are listed in Table 2). The convection zone entropy was estimated from data kindly supplied by J. Christensen-Dalsgaard (note, however, that we estimated the entropy for models [25] with a Mihalas- Hummer-Däppen equation of state, whereas these models were computed with an Eggleton-Faulkner- Flannery equation of state [41]). The arrangement of the models confirms the character of the changes described: models 6, 21, 22 have different ages and, consequently, different X profiles, and lie parallel to the δX dependence. The situation with models 7, 27, 28, having different core opacities and lying parallel to the SK dependence is similar. Effect of opacity variations localized near the convection zone base is insignificant: the changes are $\delta Y = 0.0003$, $\delta S = -0.03$, $\delta h/R_\odot = 0.012$ for opacity variation amplitude

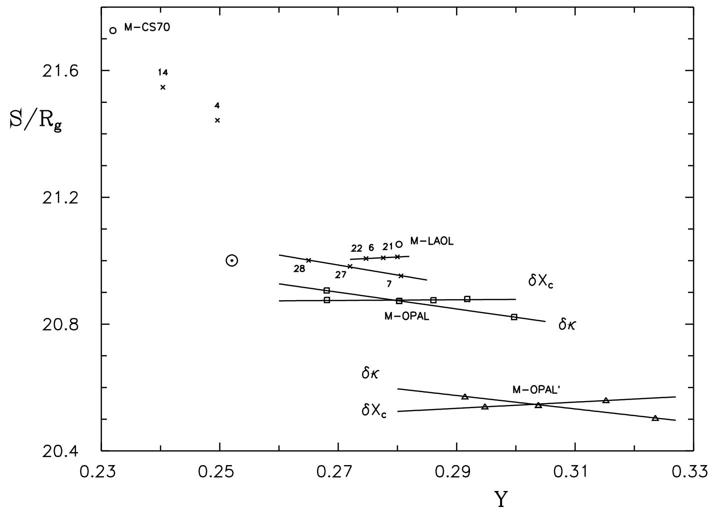

Figure 4: Specific entropy of the convective envelope S/R_g and helium abundance Y in Solar models. Circles show models calculated in this paper (Table 1), and crosses show models from [25] (numbered as in Table 2). Lines connecting squares and triangles show $S(Y)$ dependences for variations of opacity (labeled $\delta\kappa$) and chemical composition (δX) in the model core. Transition from M-OPAL to M-OPAL' corresponds to change of the radiative zone opacity. The Solar symbol shows the helioseismic determination of helium abundance and entropy in the Solar envelope.

$a_\kappa = 0.3$.

The above result brings us to formulate the following suggestion. Solar models with a given opacity in the convection zone (as well as equation of state and reaction rates) will have the same entropy for a given helium abundance, i.e., will belong to the same $S(Y)$ dependence, irrespective of their chemical composition profile (either standard evolutionary profile or not) and core opacity. If we restrict ourselves to a comparison of entropies S/R_g of models with accuracy $0.1 mol/g$ and the interval $Y \in [0.23 - 0.28]$, the above assertion means that all models with a given opacity in the radiative zone have the same entropy in the convective envelope.

CONCLUSION

Use of the specific entropy of the Solar envelope as a model parameter allows us to consistently follow the influence of changes in physical ideas on the model structure. At present, there are estimates for parameters of the region of the second helium ionization in the convection zone, based on oscillation data [42] (see also [43]). These estimates make it possible to obtain the helium abundance and entropy of the convective envelope independent of assumptions about the structure of the Solar core. To bring the theoretical spectrum of oscillations into agreement with the observed frequencies, agreement of these model parameters is necessary. The circled dot in Fig. 4 corresponds to helioseismic estimates obtained for estimation of entropy for the optimum model from [42]. It is clear that all the standard models considered are rather far from the helioseismic estimate. Using Fig. 4, we can estimate the possibility of bringing the Solar model parameters into agreement. This may be done either by means of a substantial reduction of core opacities, proceeding from the OPAL model, or by a considerable decrease of hydrogen core abundance (by more than 20%) with simultaneous reduction of opacities, proceeding from the LAOL model (see also the models with reduced core opacities from [44]). Such models will have a low flux of chlorine neutrinos. However, judgements about the necessity of opacity changes make sense only for the many models considered with evolutionary or nearly evolutionary chemical abundance profiles. At the same time, from the viewpoint of consistency of parameters, models with diffusive helium sedimentation from the convection zone ([45, 46] and references therein) may be promising. In these models, a rather good consistency is achieved of helium abundance and convection zone depth with helioseismic determinations of these quantities using the OPAL opacities [45]. The question of the consistency of the entropy remains open, and it is possible that its value in the models is within the limits of helioseismic calibration errors.

In addition, the relationships considered may be somehow affected by the assumed nuclear reaction cross-sections and equation of state, although considerable variations of these quantities are hardly admissible physically. In any case, in analysis of the Solar models obtained, it is necessary to use both parameters - entropy and helium abundance.

ACKNOWLEDGMENTS

The authors thank W.Däppen for supplementary calculations of the entropy in the framework of the Mihalas- Hummer-Däppen equation of state. The authors are also grateful to J. Christensen-Dalsgaard for additional data on the models in [25].

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